A note on forecasting exchange rates using a cluster technique

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Abstract: In this note, we propose a cluster method as a simple predictive tool to forecast exchange rates (specifically the Japanese Yen and the British Pound against the US Dollar). The general goal in this study is two-fold. First of all, we verify whether or not we can accurately predict the exchange rate movements using the suggested method. Secondly, we use the generated sign predictions to build a simple trading strategy and check if we can obtain above-normal profits in the foreign exchange market. Our results reveal a sign forecasting ability for one-period-ahead which is lost when more periods ahead are considered. On the other hand, our simple trading strategy does not obtain above-normal profits.

Keywords: cluster forecasting method; exchange rate forecasting; foreign exchange market; trading strategies.


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1 Introduction

The use of technical analysis in the foreign exchange market is a common practice assumed by different agents such as speculators, multinationals, governments and international investors. The historical data is considered useful for these agents in order to analyse and anticipate exchange rate movements. However, its widespread use in financial markets collides with the traditional academic point of view. Specifically, academics consider that, firstly, the exchange rates follow a random walk process. Therefore, the best exchange rate predictor can be obtained considering exclusively the previous value and, in consequence, exchange rates returns are practically unpredictable. Many authors have empirically demonstrated that foreign exchange rates, just like other
A note on forecasting exchange rates using a cluster technique

69

financial time series, are well approximated by a random walk model. For example, Meese and Rogoff (1983) realised a competition among different models and they found that a naïve random walk model showed the highest predictive ability. This result was updated later by the same authors, and the conclusions did not change (Meese and Rogoff, 1988). Secondly, academics also argue that no profitable trading strategies can be implemented to obtain any extraordinary and persistent profit from a trading rule which decided when to buy or when to sell a currency based exclusively on past values of the exchange rate.

In spite of these academic considerations, nowadays it is widely known that exchange rates show a non-linear dynamic which could be exploited to improve the predictions obtained by the random walk model and, moreover, these predictions could be employed to articulate profitable strategies (Tenti, 1996). Many forecasting attempts have been done employing different non-linear forecasting methods such as weighted local regression (Diebold and Nason, 1990), local regression (Álvarez-Díaz and Álvarez, 2008; Álvarez-Díaz, 2008b), neural networks (Álvarez-Díaz and Álvarez, 2005), genetic programming (Álvarez-Díaz and Álvarez, 2003) or, more recently, data fusion (Álvarez-Díaz and Álvarez, 2005) and evolutionary neural networks (Álvarez-Díaz and Álvarez, 2007).

In this note, we use a novel cluster technique to anticipate the dynamic evolution of exchange rates. The method was previously applied to analyse the Spanish stock market (Amigo et al., 2004), and it was already showed its higher predictive capacity in comparison with local regression (Álvarez-Díaz, 2008a). The general objective in this study is two-fold. First of all, we verify whether or not we can accurately predict the exchange rate movements using the cluster method; therefore, we will try to predict the direction of the sign before forecasting the exact value (point prediction). From an empirical point of view, sign prediction seems much more interesting for the financial agents because even the smallest forecast errors could cause heavy losses in capital if the direction of the forecast is mistaken (Tenti, 1996; Lisi and Medio, 1997). Secondly, we use the generated sign predictions to build a simple trading strategy, and we check if we can obtain above-normal profits in the foreign market. In summary, our main goal will be to answer two questions: do exchange rates really follow a random walk? Can we obtain abnormal and persistent profits in the foreign exchange market using a cluster forecasting method?

The note is presented as follows. After this introduction, Section 2 explains the forecast method used in this study. In Section 3, certain important aspects of the forecasting exercise and the strategy simulation are commented on and the obtained results are presented. Finally, in Section 4 we draw our conclusions.

2 Cluster method

Given a time series \( \{ x_t \}_{t=1}^T \), we can construct a cluster method which anticipates its dynamic evolution. Specifically, the method will try to determine whether the time series will go up or go down (an appreciation or depreciation in exchange rates terminology) \( \tau \) periods ahead. The procedure can be easily explained considering a series of simple
steps. In the first stage, the trajectory matrix \( M_{T-m+1,m} \) is constructed from the time series \( \{x_t\}_{t=1}^T \)

\[
M_{T-m+1,m} = \begin{pmatrix}
M^1 \\
M^2 \\
\vdots \\
M^{T-m+1}
\end{pmatrix}
= \begin{pmatrix}
x_1 & x_2 & \ldots & x_m \\
x_2 & x_3 & \ldots & x_{m+1} \\
\vdots & \vdots & \ddots & \vdots \\
x_{T-m+1} & x_{T-m+2} & \ldots & x_T
\end{pmatrix}
\tag{1}
\]

Each row of the trajectory matrix is made up of vectors of the following form

\[
M^i = (x_{i}, x_{i+1}, \ldots, x_{i+m-1})
\tag{2}
\]

which belongs to a vector space whose dimension \((m)\) is denoted as embedding dimension. According to the Takens’ Theorem (Takens, 1981), the geometrical trajectory of this sequence of vectors forms a multi-dimensional object at \( \mathbb{R}^m \) which maintains unaltered certain characteristics of the true but unknown dynamic system that generates the data. Furthermore, if the time series is deterministic, the Theorem guarantees the possibility of predicting its evolution using past values of the time series.

Next, the value to which each of the \( M^i \) vectors has evolved \((∀ i = 1, \ldots, T-m-\tau+1)\). To do so, the evolution matrix is constructed:

\[
E_{(T-m-\tau+1)\times(m+1)} = \begin{pmatrix}
M^1 & R^1 \\
M^2 & R^2 \\
\vdots & \vdots \\
M^{T-m-\tau+1} & R^{T-m-\tau+1}
\end{pmatrix}
= \begin{pmatrix}
x_1 & x_2 & \ldots & x_m & x_{m+\tau} \\
x_2 & x_3 & \ldots & x_{m+1} & x_{m+1+\tau} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
x_{T-m+1} & x_{T-m+2} & \ldots & x_T & x_T
\end{pmatrix}
\tag{3}
\]

where the last column shows the value generated by the vector \( M^i \) \(\tau\) periods in the future. For example, the vector \( M^1 = (x_1, x_2, \ldots, x_m) \) has generated a future value \( R^1\) \(x_m + \tau\) and the vector \( M^{T-m-\tau+1} = (x_T - m - \tau + 1, x_T - m - \tau + 2, \ldots, x_T - \tau) \) has given a future value \( R^{T-m-\tau+1} = x_T \). From this matrix \( E \), the vectors are grouped according to the sign of \( R^i \) \((∀ i = 1, \ldots, T-m-\tau+1)\). As such, the following sub-matrices are obtained:

\[
E^+ = \left\{ M^i / R^i > 0, \forall i = 1, \ldots, T-m-\tau+1 \right\}
\tag{4}
\]

\[
E^- = \left\{ M^i / R^i < 0, \forall i = 1, \ldots, T-m-\tau+1 \right\}
\tag{5}
\]

where all vectors are grouped depending on the sign of their evolution \(\tau\) periods ahead. Once the cluster has been carried out, the average vectors are calculated for each sub
matrix, obtaining in this way the centroids of the submatrices \( E^+ \) and \( E^- \) (\( \bar{C}^+ \) and \( \bar{C}^- \), respectively).

Next, the Euclidean distance between the vector \( M^{T-m+1} \) and the centeroid associated to sub matrix \( E^+ \) is calculated:

\[
d^+ = \text{dist}(\bar{C}^+, M^{T-m+1}) = \|\bar{C}^+ - M^{T-m+1}\|
\]

(6)

and the same calculation is realised in the case of the centeroid corresponding sub matrix \( E^- \):

\[
d^- = \text{dist}(\bar{C}^-, M^{T-m+1}) = \|\bar{C}^- - M^{T-m+1}\|
\]

(7)

Once we have arrived at this point of the analysis, the degree to which the vector \( M^{T-m+1} \) belongs to the sub matrix \( E^+ \) can be calculated:

\[
\mu^+ = \frac{1}{d^+ + \frac{1}{d^-}}
\]

(8)

and to the sub matrix \( E^- \):

\[
\mu^- = \frac{1}{d^- + \frac{1}{d^+}}
\]

(9)

The prediction criterion adopted for our exercise is to consider that the vector \( M^{T-m+1} = (x_{T-m+1}, x_{T-m+2}, \ldots, x_T) \) will evolve towards an appreciation \( \tau \) periods in the future if \( \mu^+ > \mu^- \); obtaining, in this particular case, a predicted value of \( \hat{R}_{T+\tau} = 1 \).

Conversely, if \( \mu^- < \mu^+ \), then a depreciation will be anticipated \( \tau \) periods ahead, and \( \hat{R}_{T+\tau} = -1 \) will be predicted.

Finally, additional technical comments have to be mentioned before running the cluster technique. Firstly, a crucial aspect using the cluster technique, just like other financial forecasting methods, is to determine appropriately the embedding dimension \( (m) \) because the success of the prediction depends to a great extent on the right choice of its value. Following the recommendation of Casdagli (1989), we will choose the embedding dimension which optimises a given fit criterion, considering exclusively a specific sub-sample. The only choice of one technical parameter represents an advantage regarding other techniques like the nearest neighbour method where the researcher also has to select the optimum number of neighbours. Secondly, the fit criterion which is employed in this study will be the success ratio defined by the expression

\[
SR = \frac{\sum_{m=1}^{M} \theta[x_t \cdot \hat{x}_t > 0]}{T}
\]

(10)
where \( SR \) is the ratio of correctly predicted signs (success ratio), \( x_t \) is the observed value, \( \hat{x}_t \) is the predicted value, \( \theta(\cdot) \) is the Heaviside function (\( \theta(\cdot) = 1 \) if \( x_t \cdot \hat{x}_t > 0 \) and \( \theta(\cdot) = 0 \) if \( x_t \cdot \hat{x}_t < 0 \)), and \( T \) is the total number of observations in the considered sample.

3 Results

The database employed is composed of weekly exchange rates data of Japanese Yen and British Pound against the US Dollar and, as usual in exchange rates forecasting, we consider the difference of the exchange rate logarithm,

\[
x_t = \log(y_t) - \log(y_{t-1})
\]

where \( y_t \) is the exchange rate under analysis, \( \log(y_t) \) is its logarithmic transformation and \( x_t \) is its return. If the exchange rates followed a random walk, the sequence \( \{x_t\}_{t=1}^{T} \) would be random and, in consequence, unpredictable.

The sample period goes from the first week of January 1973 to the last week of July 2002, comprising a total of 1541 observations. In order to achieve a fair predictive exercise, the total sample was divided in three sub-periods: training, selection and out-of-sample. The first one, composed by the first 1080 observations, is reserved as history of the time series. The selection period, which covers the 306 following observations, is used to determine the optimal embedding dimension and, finally, we have reserved the last 155 observations to validate the predictive ability of the cluster technique.

Figure 1 shows the sensitivity of the cluster method in the selection period considering different embeddings. As we can observe, both series show a certain forecasting stability. However, as mentioned before, we have chosen the embedding dimension \( m \) which provides the highest \( SR \) in the selection period. Once we have determined the embedding dimension, we use this information to adjust our cluster method and obtain the out-of-sample predictions. Table 1 provides the one-period-ahead \( SR \) for both currencies. We can observe how the results obtained are very similar for both exchange rates (57.7 and 58.7 for the Yen/$ and BP/$, respectively). This fact confirms the belief in exchange rates forecasting that there is little variation in results from one exchange rate to another when weekly data is used (Diebold and Nason, 1990). In order to verify whether or not the out-of-sample accuracy is statistically significant, we have applied the traditional test proposed by Pesaran and Timmermann (1992) (PT test, hereinafter) and a test based on the method of surrogate data (Theiler et al., 1992). The PT test verifies whether the percentages of successes obtained with the cluster methods differ significantly from those that would be obtained if the real \( (x_t) \) and predicted \( (\hat{x}_t) \) values were independent. The test, under the null hypothesis of independence, shows a normal standard distribution. Therefore, the critical values with a significance level of 1%, 5% and 10% correspond to the values 2.33, 1.645 and 1.282, respectively. On the other hand, employing the surrogate method, we generate artificially 1.000 time series randomly shuffling the original time series \( \{x_t\}_{t=1}^{T} \). By scrambling the data, any possible deterministic structure should be destroyed maintaining the distributional properties of the original series.
A note on forecasting exchange rates using a cluster technique

Figure 1  Selection of the embedding dimension (see online version for colours)
Figure 1  Selection of the embedding dimension (see online version for colours) (continued)
Later on, we apply the cluster forecasting method on these artificial series; we calculate their corresponding SR and, finally, we construct the SR empirical distribution. If there was no forecasting capacity, the SR obtained in the original series should not be statistically different than the SR obtained by the shuffled series. Using the empirical distribution of SR, we can build a confidence interval with a specific significant level, in our case at the 95%. Any SR inside the empirical interval would be considered as the result of the application of the method on random signals.

Table 1  
Cluster forecasting method: results one-period ahead

<table>
<thead>
<tr>
<th>Exchange rates</th>
<th>Embedding dimension (m)</th>
<th>Selection period</th>
<th>Out-of-sample period</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Success ratio (%)</td>
<td>Empirical confidence interval</td>
<td>Success ratio (%)</td>
</tr>
<tr>
<td>Yen/$</td>
<td>9</td>
<td>59.48, 2.85 (43.1, 56.8)</td>
<td>57.7, 1.83 (43.6, 56.4)</td>
</tr>
<tr>
<td>British Pound/$</td>
<td>2</td>
<td>54.9, 1.62 (46.7, 53.3)</td>
<td>58.7, 2.2 (47.1, 54.2)</td>
</tr>
</tbody>
</table>

Observing again Table 1 and, specifically, the PT test and the empirical interval, we can affirm that the sign prediction obtained by the cluster method differs significantly from the 50%, expected success ratio if the exchange rate returns were independent and unpredictable. Therefore, we can already answer the first question expressed in the introductory section: at least in the short run (one-period-ahead), there exist evidences against the belief that the exchange rates follow a random walk and returns are unpredictable. However, can we maintain this predictive ability when different forecasting periods ahead are considered? In Figure 2 we find out the possibility of nonlinear dependence for different forecasting periods. We observe how the most accurate predictions is achieved for one period ahead and, for more periods ahead, the out-of-sample SR decreases and fluctuates around 50%. Therefore, at least for the considered periods, the results seem to indicate the only existence of a short-term predictable structure in the exchange rates dynamic.

Regarding the strategy simulation setup, we employ the predictions obtained by the cluster method to articulate a simple trading strategy, and we verify in a blind experiment what it would have happened if we had invested one dollar (back-testing procedure). The back-testing period covers from 28/07/1999 to 31/07/2002 (out-of-sample period). An additional issue is how to build our trading strategies. Constructing a strategy requires the use of statistically relevant predictions. As we can observe in Figure 3, only relevant predictions are obtained for one-period ahead in the Selection Period. Therefore, the buying and selling orders of our trading strategy will be exclusively based on the following if-statements:

If \( \hat{x}_{t+1} < 0 \), then buy the currency

If \( \hat{x}_{t+1} < 0 \), then sell the currency

On the other hand, we assume in our exercise that the investor is risk-averse when obtain profits and risk-seeking when she obtains losses (Tversky, 1990). However, we also verified in our study that assuming risk neutrality does not substantially modify the results. Finally, we also assume that the investment will be small enough to avoid substantial modifications of the market conditions (no reflexivity phenomenon).
Figure 2  Out-of-sample sign prediction $τ$ periods ahead (see online version for colours)
Figure 2  Out-of-sample sign prediction $\tau$ periods ahead (see online version for colours) (continued)
Figure 3  Selection period sign prediction \( \tau \) periods ahead (see online version for colours)
Figure 3  Selection period sign prediction $\tau$ periods ahead (see online version for colours)
(continued)
Once the strategy set-up is defined, we apply our *if-statements* during the back-testing period. Table 2 comprises and summarises the trading strategy results. At the end of the back-testing period, the strategies based on our predictions do not obtain significant positive profits for both currencies (–3.65 and 0.81 Cents for the Yen/$ and the BP/$, respectively). However, we should also take into account that we have employed a very simple ‘if-statements’ to generate our trading strategy. Perhaps the incorporation of more ‘if-statements’ from other forecasting methods (confirmation method) and/or increasing their complexity would allow obtaining positive profits. Therefore, additional research must be done in order to obtain more accurate predictions and construct more complex trading strategies.

### Table 2

<table>
<thead>
<tr>
<th>Transaction</th>
<th>Buy order</th>
<th></th>
<th></th>
<th>Sell order</th>
<th></th>
<th></th>
<th>Profit (¢)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Date</td>
<td>Price (¥/$)</td>
<td>Date</td>
<td>Price (¥/$)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>4/8/99</td>
<td>114.5</td>
<td>17/11/99</td>
<td>105.6</td>
<td>8.41</td>
<td></td>
<td></td>
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<tr>
<td>2</td>
<td>24/11/99</td>
<td>104.0</td>
<td>15/12/99</td>
<td>103.3</td>
<td>0.68</td>
<td></td>
<td></td>
</tr>
<tr>
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<td>24/11/99</td>
<td>0.619</td>
<td>–12.73</td>
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<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>–3.65</td>
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</tbody>
</table>

(a)

<table>
<thead>
<tr>
<th>Transaction</th>
<th>Buy order</th>
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<th></th>
<th>Sell order</th>
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</thead>
<tbody>
<tr>
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<td>Date</td>
<td>Price (¥/$)</td>
<td>Date</td>
<td>Price (¥/$)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1/9/99</td>
<td>0.62</td>
<td>15/9/99</td>
<td>0.619</td>
<td>0.81</td>
<td></td>
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<tr>
<td>2</td>
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<td>27/10/99</td>
<td>0.606</td>
<td>0.92</td>
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</tr>
<tr>
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<td>8/12/99</td>
<td>0.616</td>
<td>26/1/00</td>
<td>0.61</td>
<td>0.98</td>
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<tr>
<td>4</td>
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<td>0.624</td>
<td>24/7/02</td>
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<td>–1.9</td>
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<td></td>
<td></td>
<td>0.81</td>
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</tr>
</tbody>
</table>

(b)

### 4 Conclusions

In this note we have applied a novel forecasting procedure based on clustering data in order to forecast the exchange rates evolution. The predictive results seem to indicate the existence of a statistically significant short-term predictable structure in the exchange rates dynamic. However, when analysing the economic value of the prediction, our strategy based on simple ‘if-statements’ does not allow obtaining positive profits. Nevertheless, this conclusion must be qualified. Future research could allow us even more accurate predictions and/or improve the economic results of the trading strategies.
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References


